# Q1 T1 — Triangle Report (step-by-step)

Given triangle vertices (orthonormal frame):

A = (4, 4), B = (-6, -1), C = (-2, 4)

We label sides in the usual way: a = |BC|, b = |AC|, c = |AB|.

## 1) Side vectors and lengths

Vector AB = B - A = (-10, -5)

Vector BC = C - B = (4, 5)

Vector AC = C - A = (-6, 0)

Side lengths (exact forms and decimals):

a = |BC| = sqrt(41) ≈ 6.403124

b = |AC| = 6 (exact)

c = |AB| = sqrt(125) = 5\*sqrt(5) ≈ 11.180340

Perimeter P = a + b + c ≈ 23.583464

Area (using cross product of AB and AC): |AB x AC| / 2 = 15.000000

Work: AB x AC = -30 → area = 1/2 \* |...| = 15

## 2) Angles (using scalar/dot product)

Angle at A (between AB and AC):

cos(A) = (AB·AC)/(|AB||AC|) = 0.894427 → A ≈ 26.565051°

Angle at B (between BA and BC):

cos(B) = (BA·BC)/(|BA||BC|) = 0.907959 → B ≈ 24.775141°

Angle at C (between CA and CB):

cos(C) = (CA·CB)/(|CA||CB|) = -0.624695 → C ≈ 128.659808°

Check: A + B + C ≈ 180.000000° (≈ 180°)

## 3) Eight different equations of side AC (side b)

Because A(4,4) and C(-2,4) have the same y-coordinate, AC is horizontal at y = 4.

1. Explicit (slope-intercept): y = 4

2. Implicit (standard): 0·x + 1·y - 4 = 0 (i.e. y - 4 = 0)

3. Point-normal form (using normal n = (0,1)): n·(x - A) = 0 → (y - 4) = 0

4. Vector form: r(t) = A + t(C - A) = (4,4) + t(-6,0), t ∈ ℝ

5. Parametric (segment parameter t): x = 4 - 6t, y = 4, t ∈ [0,1] (for the segment)

6. Two-point form: (y - 4)/(4 - 4) = (x - 4)/(-2 - 4) — simplifies to y = 4 (since denominator zero in y-part)

7. Intercept/normal form (unit normal n̂=(0,1)): n̂·(x,y) = 4 → 0·x + 1·y = 4

8. Distance form: distance((x,y), AC) = |y - 4|, the locus (line) is where this distance = 0

## 4) Analytical equations: r\_b (perp bisector of AC), l\_b (angle bisector at B), M\_b (midpoint), m\_b (median)

Midpoint M\_b of AC: M\_b = ((4 + (-2))/2, (4 + 4)/2) = (1.0, 4.0)

Median m\_b: line from B to M\_b.

Parametric (m\_b): r(t) = B + t(M\_b - B) = (-6,-1) + t(7,5)

Explicit (slope-intercept): y + 1 = (5/7)(x + 6) → y = (5/7)x + 23/7

Implicit: 5x - 7y + - (5\*(-6) - 7\*(-1)) simplified → 5x - 7y - 23 = 0 (check by rearranging)

Perpendicular bisector r\_b of AC:

Since AC is horizontal, r\_b is the vertical line through midpoint x = 1.

Explicit: x = 1

Implicit: 1·x + 0·y - 1 = 0

Parametric: (1, t), t ∈ ℝ

Angle bisector l\_b at vertex B (internal):

Unit vectors along BA and BC:

u\_BA = (0.8944271909999159, 0.4472135954999579)

u\_BC = (0.6246950475544243, 0.7808688094430304)

Bisector direction vector (unnormalized): u\_BA + u\_BC = (1.5191222385543401, 1.2280824049429884)

Parametric (l\_b): r(t) = B + t \* (u\_BA + u\_BC)

Numerical explicit (approx): y = (0.808416) x + (3.850495)

Numerical implicit (approx): 0.808416 x - y + (3.850495) = 0

## 5) Lengths: h\_b (altitude from B), l\_b (internal bisector from B), m\_b (median from B)

h\_b (altitude to AC): distance from B to line y=4 → |y\_B - 4| = 5.000000

Derivation: AC horizontal ⇒ altitude from B is vertical distance.

m\_b (median length) = distance(B, M\_b) = |B - M\_b| = 8.602325

Work: vector B->M\_b = (7.0, 5.0)

l\_b (internal angle bisector length) via formula l\_b^2 = a·c·(1 - b^2/(a+c)^2):

Using a = 6.4, b = 6, c = 11.1 → l\_b ≈ 7.9

## 6) Notable points: Orthocenter H, Centroid G, Circumcenter O, Incenter I

Orthocenter H (intersection of altitudes) = (-6, 12.0)

Centroid G (average of vertices) = (-1.3, 2.3)

Circumcenter O (intersection of perp. bisectors) = (1.0, -2.5)

Incenter I (weighted average by opposite side lengths) = (-1.3, 2.7)

Check vector relation: vector(HG) = 2 \* vector(GO)

vector(HG) = G - H = (4.6, -9.6)

vector(GO) = O - G = (2.3, -4.8)

Is HG = 2·GO? → True

## 7) Incircle and Circumcircle — centers, radii and equations

Circumcircle:

Center O = (1.0, -2.5)

Circumradius R = 7.158911 (exact R^2 = 51.250000)

Canonical (standard) equation:

(x - 1.000000)^2 + (y - (-2.500000))^2 = 51.250000

Parametric:

x(θ) = 1.000000 + 7.158911 cos θ, y(θ) = -2.500000 + 7.158911 sin θ

Polar (centered at O): r = R (i.e., distance from O is constant).

Incircle:

Center I = (-1.38860782503305, 2.7279222492049073)

Inradius r = 1.272078

Canonical equation:

(x - -1.388608)^2 + (y - (2.727922))^2 = 1.618182

Parametric:

x(θ) = -1.388608 + 1.272078 cos θ, y(θ) = 2.727922 + 1.272078 sin θ

Polar (centered at I): r = r\_in (distance from I is constant).

## 8) Area and perimeter (circumference) of circles

Circumcircle:

Area = π R^2 ≈ 161.006623

Circumference = 2 π R ≈ 44.980761

Incircle:

Area = π r^2 ≈ 5.083668

Circumference = 2 π r ≈ 7.992700

## 9) GeoGebra plotting (place for screenshot)

Use 2D GeoGebra to plot the triangle with:

- triangle A(4,4), B(-6,-1), C(-2,4)

- incircle and circumcircle (use centers and radii above)

- median m\_b (B to midpoint of AC), altitude h\_b (from B), angle bisector l\_b (from B), perpendicular bisector r\_b

Insert your GeoGebra screenshot here (you requested space to add screenshots).